DETERMINATION OF WAVE NOISE SOURCES USING SPECTRAL PARAMETRIC MODELING

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ABSTRACT

We measure transistor noise power density and compute the Fourier's transform. Finally, spectral parametric modeling is used to extract noise waves correlation matrix. Results obtained by this new method has been experimentally compared with a conventional method.

INTRODUCTION

This new method is based on the analysis of the wave power density in the time domain, which is obtained by a Fourier's transform of the measured noise power. Thus, we obtain the noise power distribution in the time domain called wave signal autocorrelation. This point of view allows the use of spectral estimators (parametric models) like autoregressive (AR) model. Only two noise power measurements corresponding to two different input coefficients are necessary to extract the wave correlation matrix.

THE NOISE WAVE REPRESENTATION

This kind of representation is very helpful for the microwave domain analysis. In the noise wave representation, a circuit element's noise is described by using waves that emanates from its ports. A linear two-port represented by noise wave and scattering parameters is shown in Fig. 1. Noise waves b_{N1} and b_{N2} contribute to the scattered waves. Thus, the wave variables and scattering parameters satisfy:

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \cdot \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} b_{N1} \\ b_{N2} \end{pmatrix} \quad \textbf{(1)}$$

The noise waves are time-varying complex random variables characterised by a correlation matrix given by [4]:

$$C_S = \begin{pmatrix} |\langle b_{N1}\rangle|^2 & \langle b_{N1} b_{N2}^*\rangle \\ \langle b_{N1}^* b_{N2}\rangle & |\langle b_{N2}\rangle|^2 \end{pmatrix} = \begin{pmatrix} |\langle b_{N1}\rangle|^2 & C S_{21}^* \\ C^* S_{21} & |\langle b_{N2}\rangle|^2 \end{pmatrix} \quad \textbf{(2)}$$

where $\langle . \rangle$ indicates time averaging with an implicit assumption of ergodicity and jointly wide-sense stationary processes. The diagonal terms of C_S give the noise power deliverable to the terminations in a 1-Hz bandwidth and the off-diagonal terms are correlation products. All noise power are normalized with regard to kT_0 factor where k is the Boltzmann's constant and T_0 is equal to 273 K.

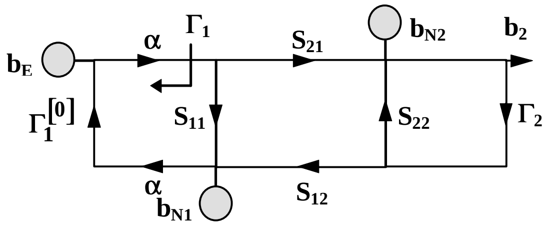

Fig. 1: The representation of a two-port circuit element using scattering parameters and noise waves

The output noise power is equal to $\langle |b_2|^2 \rangle$ and its expression is given by:

$$\frac{\langle |b_2|^2\rangle}{|S_{21}|^2} = \frac{\langle |b_E|^2\rangle + |\Gamma_1|^2 \langle |b_{N1}|^2\rangle}{|(1-\Gamma_1 S_{11})(1-\Gamma_2 S_{22}) - \Gamma_1\Gamma_2 S_{12} S_{21}|^2} + \frac{2\,\mathrm{Re}\left[\Gamma_1 (1-\Gamma_1 S_{11})^* \left\langle b_{N1} \left(\frac{b_{N2}}{S_{21}}\right)^* \right\rangle\right]}{|(1-\Gamma_1 S_{11})(1-\Gamma_2 S_{22}) - \Gamma_1\Gamma_2 S_{12} S_{21}|^2} + \frac{|1-\Gamma_1 S_{11}|^2 \left\langle \left|\frac{b_{N2}}{S_{21}}\right|^2 \right\rangle}{|(1-\Gamma_1 S_{11})(1-\Gamma_2 S_{22}) - \Gamma_1\Gamma_2 S_{12} S_{21}|^2} \quad \textbf{(3)}$$

NOISE RECEIVER MODELING

The RF signal emanating from the device under test (DUT) is amplified through a 20 dB amplifier (Fig. 2). Then, the amplified signal is measured by the HP 8970B noise figure meter (noise power measurement mode) in single side band. For this noise receiver, the standard noise parameters [4][5] can be simplified with the assumption of uncorrelated noise wave sources. Our study has proven that $\left\langle |b_{N1}^{NR}|^2 \right\rangle$ (NR: Noise Receiver) effect can be neglected. Thus:

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$$r_n^{NR} \approx \frac{1}{4} \cdot \left\langle \left| \frac{b_{N2}^{NR}}{S_{21}^{NR}} \right|^2 \right\rangle \cdot \left| 1 + S_{11}^{NR} \right|^2 \quad \textbf{(4)}$$

$$F_{min}^{NR} \approx 1 + \left\langle \left| \frac{b_{N2}^{NR}}{S_{21}^{NR}} \right|^2 \right\rangle \cdot \left[1 - \left| S_{11}^{NR} \right|^2 \right] \quad \textbf{(5)}$$

$$\Gamma_{opt}^{NR} \approx S_{11}^{NR\,*} \quad \textbf{(6)}$$

with $$F_0^{NR} = 1 + \left\langle \left| \frac{b_{N2}^{NR}}{S_{21}^{NR}} \right|^2 \right\rangle \quad \textbf{(7)}$$

Where F_0^{NR} is the noise factor under matched input impedance ($|\Gamma_1| = 0$). Equation (6) is in agreement with studies made by using conventional methods [6].

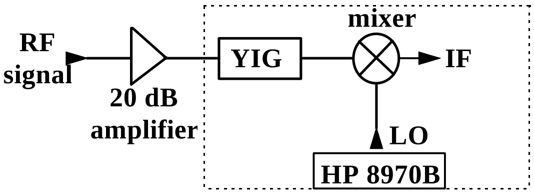

Fig. 2: The schematic representation of the noise receiver

TRANSISTOR NOISE WAVE SOURCES EXTRACTION

We have studied the output noise wave behaviors of several transistors such as MESFETs and HEMTs and in this case, $\left\langle |b_2|^2 \right\rangle$ expression in (3) is simplified as follows:

$$\frac{\left\langle |b_2|^2 \right\rangle}{|S_{21}|^2} = \frac{\left\langle |b_E|^2 \right\rangle + |\Gamma_1|^2 \left\langle |b_{N1}|^2 \right\rangle}{|1 - \Gamma_1 S_{11}|^2} + \frac{-2|\Gamma_1|^2 \operatorname{Re}(S_{11}^{*} C) + 2\operatorname{Re}(\Gamma_1 C)}{|1 - \Gamma_1 S_{11}|^2} + \left\langle \left| \frac{b_{N2}}{S_{21}} \right|^2 \right\rangle \quad \textbf{(8)}$$

After an usual calibration, two noise power measurements with different input reflection coefficients ($|\Gamma_1|$ around 0.8 and $|\Gamma_1|$ close to 0) are necessary to separate the two terms that compose (8) which are:

$$\frac{\left\langle |b_E|^2 \right\rangle + |\Gamma_1|^2 \left\langle |b_{N1}|^2 \right\rangle - 2|\Gamma_1|^2 \operatorname{Re}(S_{11}^{*} C) + 2\operatorname{Re}(\Gamma_1 C)}{|1 - \Gamma_1 S_{11}|^2} \quad \textbf{(9)}$$

and

$$\left\langle \left| \frac{b_{N2}}{S_{21}} \right|^2 \right\rangle \quad \textbf{(10)}$$

The expression in (9) can be developed in order to make the electrical length between Γ_1 and S_{11} appear. Indeed, the input coefficient reflection (Fig. 1) is supposed to be:

$$\Gamma_1 = \Gamma_1^{[0]} \alpha^2 e^{-j2\pi f\tau} = \left| \Gamma_1^{[0]} \right| \alpha^2 e^{-j2\pi f\tau + \Phi} \quad \textbf{(11)}$$

(9) has a new formulation with the knowledge of (11):

$$\gamma_y(f) = \frac{\left\langle |b_E|^2 \right\rangle + |\Gamma_1|^2 \left[\left\langle |b_{N1}|^2 \right\rangle - 2|S_{11}C| \cos(\Phi_{S_{11}} - \Phi_C) \right]}{\left| 1 - \Gamma_1^{[0]} \alpha^2 S_{11} e^{-j2\pi f\tau} \right|^2} + \frac{2|\Gamma_1 C| \cos(2\pi f\tau - \Phi - \Phi_C)}{\left| 1 - \Gamma_1^{[0]} \alpha^2 S_{11} e^{-j2\pi f\tau} \right|^2} \quad \textbf{(12)}$$

This expression can be written in a way to make a power spectral density (PSD) appear:

$$\gamma_y(f) = \frac{\gamma_x(f) \left[1 + \frac{2|\Gamma_1 C|}{\gamma_x(f)} \cos(2\pi f\tau - \Phi - \Phi_C) \right]}{\left| 1 - \Gamma_1^{[0]} \alpha^2 S_{11} e^{-j2\pi f\tau} \right|^2} \quad \textbf{(13)}$$

Where $\gamma_x(f)$ is defined by:

$$\gamma_x(f) = \left\langle |b_E|^2 \right\rangle + |\Gamma_1|^2 \left[\left\langle |b_{N1}|^2 \right\rangle - 2|S_{11}C| \cos(\Phi_{S_{11}} - \Phi_C) \right] \quad \textbf{(14)}$$

The notation γ_y (resp. γ_x) are y (resp. x) signal PSD. The PSD totally describes behaviors of stationary gaussian processes. Then we compute inverse Fourier's transform in order to obtain y and b_{N2} autocorrelation sequences.
With N γ_y measurements, each element of autocorrelation sequence is calculated [1][7]:

$$C_y[k] = \frac{1}{N} \sum_{i=0}^{N-1} \gamma_y(f_i) e^{-j2\pi \frac{ik}{N}} \text{ with } 0 \le k \le N-1 \quad \textbf{(15)}$$

Where f_i is the frequency that belongs to the frequency range measurement [f_{min}, f_{max}]. The AR modeling is a parametric PSD modeling under the assumption of signal gaussianity [7]. Before explaining AR modeling, let us introduce the reduced frequency variable ν. Fig. 3 gives equivalence between measurement frequency and reduced frequency and ν verifies:

$$\nu \in \left[0, .., \frac{i}{N}, .., \frac{N-1}{N} \right] \quad \textbf{(16)}$$

Reduced Frequency ν	Frequency f_i
0	f_{min}
k/N	$f_{min} + k f_{step}$
(N-1)/N	f_{max}

Fig. 3: Relation between reduced frequency and measurement frequency

The AR modeling decomposes the y PSD in this way:

$$\gamma_y(\nu) = \frac{\sigma^2}{\left| 1 + \sum_{k=1}^{P} a_k e^{-j2\pi\nu k} \right|^2} \quad \textbf{(17)}$$

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Where (σ^2, a_1, ..., a_P) are called autoregressive (AR) parameters and P+1 the AR order. AR parameters are extracted by solving (18), called Yule-Walker equations. It is done by using theLevinson's recursive algorithm [1][7].

$$\begin{bmatrix} C_y[0] & C_y[1] & \Lambda & C_y[P] \\ C_y[1] & C_y[0] & \Lambda & C_y[P-1] \\ C_y[2] & C_y[1] & \Lambda & C_y[P-2] \\ M & M & M & M \\ C_y[P] & C_y[P-1] & \Lambda & C_y[0] \end{bmatrix} \begin{bmatrix} 1 \\ a_1 \\ a_2 \\ M \\ a_P \end{bmatrix} = \begin{bmatrix} \sigma^2 \\ 0 \\ 0 \\ M \\ 0 \end{bmatrix} \quad \textbf{(18)}$$

With this kind of modeling, we extract the γ_x value (20). A good estimation for C(ν) in (2) is given by:

$$C(\nu) \approx -\frac{\gamma_x(\nu)}{\Gamma_1^{[0]}(\nu)} \cdot \frac{a_{2Q}}{a_Q} \quad \textbf{(19)}$$

Where $\gamma_x(\nu)$ is:

$$\gamma_x(\nu) \approx \frac{\sigma^2}{\left(1 - \left|\frac{a_{2Q}}{a_Q}\right|^2\right) \left|1 + \sum_{k=1}^{U} a_k e^{-j2\pi\nu k}\right|^2} \quad \textbf{(20)}$$

$\Gamma_1^{[0]}$ is an open and Q is the rank where $|a_k|$ has its maximum value. U is chosen to be equal to Q/2. Then, the standard noise parameters may be derived from the noise wave sources, in order to make the comparison easier with other usual extraction methods [10]. Let us introduce Γ_C and $S_{11_{EQ}}$ which simplifies standard noise parameters expression:

$$b_{N1} = b_{N1_{NC}} + \Gamma_C \frac{b_{N2}}{S_{21}} \quad \textbf{(21)}$$

where $b_{N1_{NC}}$ is the part of b_{N1} not correlated with b_{N2}. Now, we are able to define a new reflection coefficient:

$$S_{11_{EQ}} = S_{11} - \Gamma_C \quad \textbf{(22)}$$

Then, the standard noise parameters may be written [4][5]:
Normalized noise resistance:

$$r_n = \frac{1}{4} \cdot \left[\left\langle \left|\frac{b_{N2}}{S_{21}}\right|^2 \right\rangle \cdot \left|1 + S_{11_{EQ}}\right|^2 + \left\langle \left|b_{N1_{NC}}\right|^2 \right\rangle\right] \quad \textbf{(23)}$$

Optimum reflection coefficient:

$$\frac{\Gamma_{opt}}{\left|1 + \Gamma_{opt}\right|^2} = \frac{S_{11_{EQ}}^{*} \left\langle \left|b_{N2}\right|^2 \right\rangle}{\left\langle \left|b_{N2}\right|^2 \right\rangle \cdot \left|1 + S_{11_{EQ}}\right|^2 + \left|S_{21}\right|^2 \left\langle \left|b_{N1_{NC}}\right|^2 \right\rangle} \quad \textbf{(24)}$$

and therefore: $ARG(\Gamma_{opt}) = -ARG(S_{11_{EQ}})$ **(25)**

Minimum noise factor:

$$F_{min} = 1 + \left\langle \left|\frac{b_{N2}}{S_{21}}\right|^2 \right\rangle \cdot \left[1 - \left|S_{11_{EQ}}\right| \left|\Gamma_{opt}\right|\right] \quad \textbf{(26)}$$

$|\Gamma_{opt}|$ values should be derived from (24) by solving a second degree equation.

MEASUREMENTS AND COMPARISONS

The measurements are done in a 2.8-18 GHz frequency range with a 38 MHz frequency step, in this case a GEC-MARCONI 4x75 µm transistor at Vgs=-.7 V, Vds=5V and Ids=10 mA. Fig. 4 gives γ_y(f) and γ_x(f) value while Fig. 5 indicates b_{N2} PSD that is fitted by an AR modeling too. Fig. 6 gives $\langle b_{N1} b_{N2}^{*} \rangle$ extracted value.

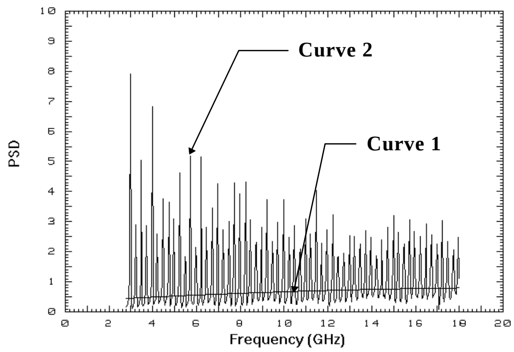

Fig. 4: ***Curve 1: $_x$ extracted value***
Curve 2: $_y$ measurement

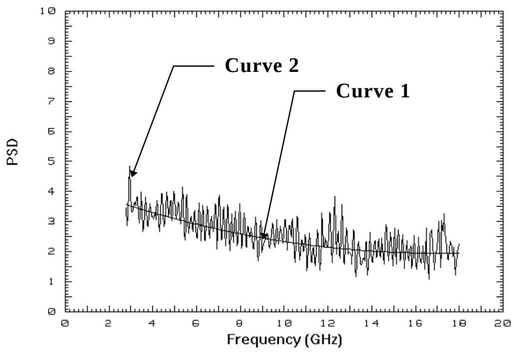

Fig. 5: ***Curve 1: b_{N2} PSD extracted value***
Curve 2: b_{N2} PSD measurement

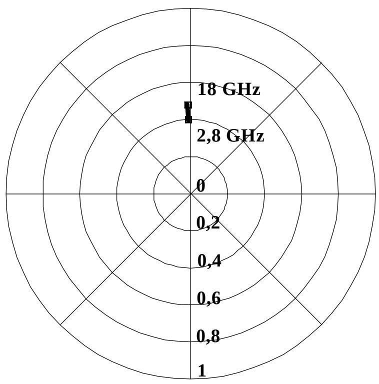

Fig. 6: ***$\langle b_{N1} b_{N2}^{*} \rangle$ extracted value from 2.8 GHz to 18 GHz***

Corresponding $\left\langle i_g i_d^* \right\rangle$ [3] value calculated with these results is mainly imaginary, respecting [8].After this extraction, the calculation of b_{N1} PSD value with the acknowledgement of b_E PSD becomes easy. Finally, comparisons are made with standard noise parameters given by GEC-MARCONI and those measured by a laboratory (Fig. 7, 8 and 9).

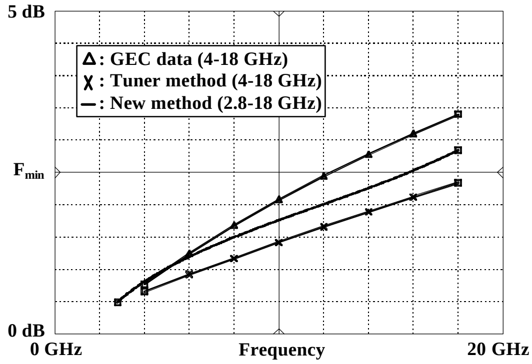

Fig. 7: F_{min} comparison

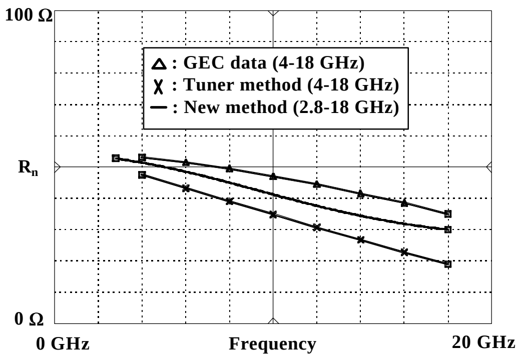

Fig. 8: R_n comparison

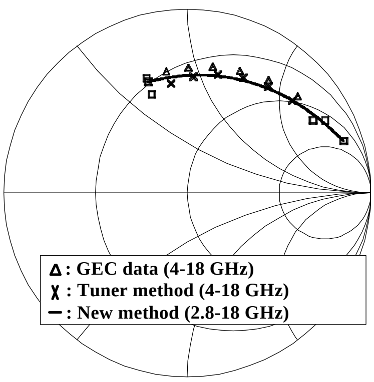

Fig. 9: $_{opt}$ comparison

CONCLUSIONS

The noise wave representation offers alternative analysis which allows time domain representation of distributed variables. The method proposed here uses a light set-up, requires no source-pull tuner and offers advantages (less time consuming ...) over conventional methods. This method will be improved in order to obtain more accurate noise sources extraction.

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